

Multifractal operators to analyse and simulate complex and intermittent systems, such as weather and climate

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The 2021 Nobel Prize in Physics was awarded jointly to Syukuro Manabe, Klaus Hasselmann and Giorgio Parisi "for revolutionary contributions to our understanding of complex physical systems". Common to these contributions is the complexity and intermittency of our planet's weather and climate [1]. With the key notion of multifractals, intermittency has become understood as resulting from an infinite hierarchy of fractal supports of the singularities of the underlying equations. However, this has been done mainly for scalar fields such as temperature and precipitation (e.g. [2-4]), whereas the velocity field that plays a major role in generating the inhomogeneity of other fields is a vector field. To solve this methodological shortcoming, we were led to introduce multifractal operators acting on vector fields [5,6]. This has revealed the crucial role of the Lie algebra of their generators. A generic case has been pointed out: that whose stochastic generators of these operators are stable Levy vectors on the Clifford algebra. This presentation refines the analysis of their structural robustness and confirms the univarsity of their statistics. Hence we push the analysis to those of their multivariate properties. This presentation also highlights the development of applications of this approach in wind energy and rainfall nowcasting.

References

- [1] D. Schertzer, C. Nicolis, Nobel Recognition for the Roles of Complexity and Intermittency, *Eos*, (July, 2022).
- [2] D. Schertzer, S. Lovejoy, Physical modeling and analysis of rain and clouds by anisotropic scaling multiplicative processes *J. Geophys. Res. AGU*, 92(D8), 9693–9714. (1987).
- [3] J. F. Royer et al., Multifractal analysis of the evolution of simulated precipitation over France in a climate scenario. *C.R. Geoscience*, 340, 431-440 (2008).
- [4] S. Lovejoy, D. Schertzer, *The Weather and Climate: Emergent Laws and Multifractal Cascades*. Cambridge U.K. Cambridge Univeristy Press (2013).
- [5] D. Schertzer, I. Tchiguirinskaia, Multifractal vector fields and stochastic Clifford algebra, *Chaos*, 25(12), 123127 (2015).
- [6] D. Schertzer, I. Tchiguirinskaia, A Century of Turbulent Cascades and the Emergence of Multifractal Operators, *Earth Space Sci.*, 7(3), 1-22 (2020).