Stochastic equations and dynamics beyond mean-field theory in Spin-Glasses

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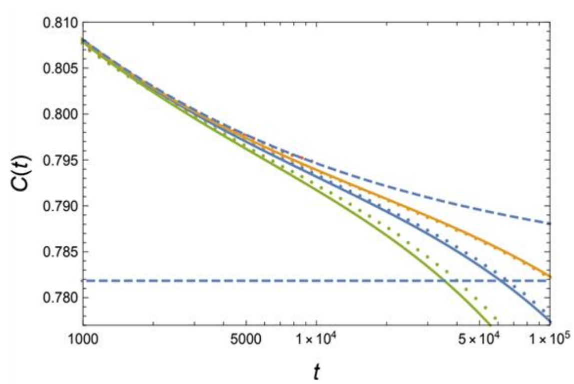
The dynamical transition occurring in spin-glass (SG) models with one step of Replica-Symmetry-Breaking (1RSB) is a mean-field artifact that disappears in finite systems and/or in finite dimensions. The dynamical transition has the features of a second-order phase transition and thus it is to be expected that the fluctuations are naturally described by a simple effective theory. Due to certain non trivial features of the corresponding theory it turns out that it is equivalent to a set of dynamical stochastic equations called stochastic-β-Relaxation (SBR) equations in [1]. Overall SBR predicts not only that the transition at the dynamical temperature is avoided or that the fluctuations deviate from mean-field theory but also that there is an essential qualitative change in the structure of the fluctuations with the appearance of dynamical heterogeneities [2-4]. We demonstrate the validity of SBR in 1RSB SG by comparing its predictions, as obtained solving numerically the stochastic equations, with Monte-Carlo simulations for the paradigmatic Ising p-spin glass model [5]. More precisely we consider a system of N spins each of which interacts with a fixed number $c = 6$ of p-spin interactions with $p = 3$ and evolve with Metropolis dynamics. The (random) lattice is such that in the large N limit loops are increasingly rare and it tends to the corresponding $c = 6$ and $p = 3$ Bethe lattice. In order to obtain parameter-free predictions we computed analytically the model-dependent parameters entering into the stochastic equations using existing and novel techniques based on the mean-field cavity method on the glassy phase of the transition. These parameters were then plugged into the code for the numerical solution of the equations. The predictions are in excellent agreement with numerical simulations for both the correlation (as seen in the figure) and its fluctuations and this demonstrates that SBR correctly describes how the sharp transition predicted by mean-field theory is turned into a crossover in 1RSB SG models with finite size. In the figure we plot the average correlation with initial equilibrium condition vs time at the dynamical temperature. Points from bottom to top are numerical data for $N = 4.5 \times 10^5$, $N = 9 \times 10^5$, $N = 1.8 \times 10^6$ (Sample numbers are respectively 9554, 8048, 7701, error bars are negligible on the scale of the plot). The data follow the Bethe lattice $N = \infty$ curve (dashed blue) at initial times and deviate from it at later times increasing with N eventually crossing the plateau value $q = 0.78184$. The solid lines are the corresponding SBR predictions describing the data when they start to deviate from the mean-field curve.



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