Universal superdiffusion of random walks on lattices with low diffusivity fractal networks

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Superdiffusive phenomena are frequently observed in systems with Lévy flights or with active motion because these are some mechanisms that enhance the transport relatively to normal diffusion. Instead, random walks on fractal media usually show subdiffusion because the self-similar distributions of obstacles restrict the transport; in these cases, the random walk dimension Dw (>2) is non-universal, in the sense that it does not depend only on the dimension Df of the fractal medium. In this work, we show that superdiffusion with universal exponents can be obtained in a random walk model on Euclidean lattices containing embedded fractal networks where the transport is slow. The model is defined with hopping rates proportional to p<<1 in the fractal sites and proportional to 1 in the remaining lattice sites. A scaling approach predicts the superdiffusive regime at short times, with a random walk dimension Df-E+2, where E is the dimension of the lattice, under the condition that the walks begin on the fractal. A crossover to the asymptotic normal diffusion is then predicted at a time of order p^{-δ}, where δ=2/(E-Df), and the anomalous root mean square displacement reaches values of order p^{-δ/2}. Numerical simulations are performed in square lattices with embedded DLAs and Sierpinski carpets (Df~1.7--1.8) and in cubic lattices with embedded percolation clusters and Menger sponges (Df~2.5--2.8). By properly avoiding finite size effects, the simulation results for several p confirm a predicted dynamic scaling relation and show that the superdiffusion can be observed in several decades of time and length for p~10^{-3} in the studied fractals. In one dimension, the model is equivalent to that of horizontal fluid infiltration in a matrix with fractal distributions of low conductivity inclusions, in which the superdiffusive scaling is expected with geological materials whose hydraulic conductivities may differ by a factor ~50, such as medium and coarse sand [Adv. Water Resour. 172, 104365 (2023)]. This work shows that superdiffusion can be obtained with a simple random walk mechanism, without long jumps or active motion, and that the random walk dimensions may be used to estimate the fractal dimensions of sets of low diffusivity or low conductivity inclusions.