On integrable parametric generalization of the Kardar-Parisi-Zhang equation of spin glasses theory and related thermodynamic stability

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Amongst the revealed new universally behaving stochastic systems we meet such important ones as directed polymers in random media and spin glasses, whose characteristic thermodynamic parameters display non-Gaussian limiting distributions and described by means of the well known Kardar-Parisi-Zhang equation:

$$\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial x^2}-\frac{u\left(\frac{\partial v}{\partial x}\right)^2}{2}:=K[v,u], \tag{1}$$

describing the distribution function of a related random variable and $u\in M_u$ is a parameter, which can often be (heuristically) computed for a particular growth model directly from the microscopic dynamics, specified by a statistical physics model under regard. As it was demonstrated in the KPZ equation well describes the long-time thermodynamics of the spin glass substrate growth owing to the competition between the surface tension smoothing forces and internal aggregation phase state, giving rise to the tendency preferentially in the direction of the local normal to the surface, represented by the corresponding nonlinear term. Thus, the following problem arises: to describe the corresponding evolution constraints

$$\frac{\partial u}{\partial t}\overset{?}{=}F[v,u,r],\quad \frac{\partial r}{\partial t}\overset{?}{=}R[v,u,r] \tag{2}$$

on a functional parameter $u\in M_u\subset C(\mathbb{R};\mathbb{R})$, which would ensue to the existence of a nontrivial hierarchy of conservation laws for the combined Langevin type evolution system

$$\left.\begin{array}{r}\partial v/\partial t=\partial(r\,\partial v/\partial x)/\partial x-u(\partial v/\partial x)^2/2\\ \partial u/\partial t=F[v,u,r]\\ \partial r/\partial t=R[v,u,r]\end{array}\right\}:=K[v,u,r], \tag{3}$$

considered as a smooth vector vield K on the combined functional manifold $M_{v,v,r}$ and which could be used for normalizing the corresponding distribution function $v\in C^2(\mathbb{R}\times\mathbb{R}_+;\mathbb{R})$ for suitably chosen initial data $v(\cdot\ ,0)\in M_v$ and supplying the related thermodynamic stability of the spin glasses growth. To solve this problem effectively, we applied the gradient-holonomic algorithm to the parametrically extended KPZ equation (1) and in the special case $r=1$ there was stated that the parametrically extended system of equations (3) reduces to evolution flow

$$\left.\begin{array}{r}v_t=v_{xx}-uv_x^2/2\\ u_t=-u_{xx}-(u^2v_x)_x/2\end{array}\right\}, \tag{4}$$

possessing an infinite hierarchy of the conserved quantities, providing thermodynamically stable spin glass growth process, being a Lax type integrable Hamiltonian system. This result, in particular, says that the parametrically extended Kardar-Parisi-Zhang equation (1) possesses a rich internal hidden symmetry, allowing to immerse it into an infinite hierarchy of Lax type completely integrable dynamical systems on a functional manifold. Moreover, it also demonstrates that the paramatrized Kardar-Parisi-Zhang equation (1) can present some interesting applications to describing thermodynamical properties of polymers structures in random media and spin glasses.

References

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