Dynamics beyond statistics and topological supersymmetry

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Supersymmetry and chaos are among the most fundamental concepts in physics and mathematics. They had been evolving almost independently until a close relation between the two was recently established within Supersymmetric Theory of Stochastic Dynamics (STS). It turned out that all stochastic differential equations (SDE) have a hidden topological supersymmetry (TS) and the spontaneous breakdown of TS is the stochastic generalization of chaos and its various manifestations such as turbulence. That is, chaos belongs to the family of the symmetry-breaking orders just as crystallization, ferromegnetism, superconductivity etc. The TS breaking order parameter represents aspects of dynamics that go beyond traditional statistical description and it has the meaning of ethereal dynamical memory known as the butterfly effect. And the Goldstone theorem explains ubiquitous long-range dynamical behavior associated with chaos and known under the umbrella term of 1/f noise.

From a more technical point of view, STS is a generalization of the Parisi-Sourlas approach to Langevin SDEs, the continuation of Ruelle theory of the generalized transfer operator, and a pseudo-Hermitian Witten-type topological field theory (TFT). Being a multidisciplinary construction it naturally sets a stage for potentially fruitful crossfertilization between a few disciplines. For example, one of the low hanging fruits of STS is the basic classification of stochastic dynamics into three major categories: thermal equilibrium, instantonic or noise-induced chaos, and ordinary chaos.

In this talk, the current state of STS and its application to neurodynamics (ND) in networks of type-I neurons will be discussed. It will be shown that the basic classification of stochastic dynamics by STS has a very natural interpretation on the side of ND. The theoretical picture from STS will be supported by numerical results on a toy model of ND and experimental data from emulation of ND on neuromoprhic hardware (Spikey, Heidelberg Univ.) and clinical data (UCLA medical school).