Random field Ising model for random single vertex origami

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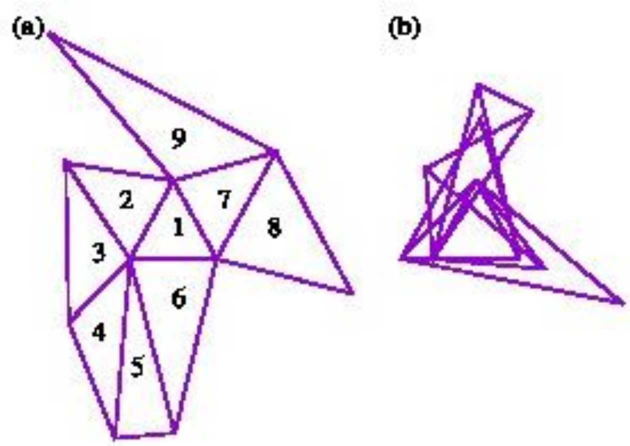
Flat-foldability problem of origami diagrams, including stamp-folding and map-folding as special cases, is a deterministic problem that asks whether it is possible to fold flat to satisfy the random allocation of mountains and valleys in the creases. These are kinds of combinatorial optimization problem and have been studied extensively in the past. For example, the stamp folding problem is known to be NP-hard to determine the maximum number of facets to be inserted, and the map folding problem is also known to be NP- hard. The classification of the computational complexity class is given by the worst-case complexity of the hardest problems in a set of various instances. On the other hand, the average-case complexity of ensembles of many instances is also interesting. The average-case complexity and the worst-case complexity sometimes exhibit different behaviors. In this research, we approach the behavior of the average-case complexity of problems related to origami. We investigate the phase transition phenomena of the flat-foldability of single vertex origami to approach better understanding of the average-case complexity of the flat foldability problem. Average behavior on the ensemble of instances of origami diagram under controlled density of imposition for folding manner, mountain or valley, of creases is observed and the thresholding value of density is obtained. We treat the problem of determining flat-foldability of the origami diagram with mapping onto a spin glass model on random graphs. A spin variable are assigned for the layer-ordering of each pair of facets which have an overlap in the pre-folded diagram. To avoid some infeasible layer-orderings some constraints are introduced. They are described as interaction terms which contain the product of two or four spin variables, while capturing the geometrical characteristics of how facets overlap. The constraints on how to overlap are classified into the following three types. 1. An intrusion of a facet into a crease which connects other two facets. 2. Some ordering among four facets connected to each sides of the two creases which are in geometrically co- incidental position the pre-folded diagram. 3. A cyclic ordering of three facets which have areas shared with all each other in the pre-folded diagram. The flat-foldability of the diagram is closely related to the (non-)existence of frustrated loops or subgraphs on the spin model with the interactions on the random (hyper)graph. In addition, a well ordered treatment of the local-constraints with three different types reveals some collective variables, cluster spins, which reflect the locked layer-orderings of some facets generated from the cooperative work of the local constraints, which lead an efficient computation of the problem. For example, the origami diagrams shown in Figs. 1(a) and (b) are originally described by 36 spin variables, but it is found that they can be described by 11 variables by translating to cluster spin.



Cluster spin	Set of elemental spins $s_{i,j}$ included
$\mathscr{C}_1 = 1$	$\{s_{1,2}, -s_{1,3}, -s_{1,4}, -s_{1,5}, -s_{1,6}, -s_{1,7}, -s_{1,8}, s_{1,9}, -s_{2,3}, -s_{2,4}, -s_{2,5}, -s_{2,6}, -s_{2,7}, -s_{2,8}, s_{2,9}, s_{3,4}, s_{3,5}, s_{3,6}, s_{3,7}, s_{3,8}, s_{3,9}, s_{4,7}, s_{4,8}, s_{4,9}, s_{5,7}, s_{5,8}, s_{5,9}, s_{6,7}, s_{6,8}, s_{6,9}, -s_{7,8}, s_{7,9}, s_{8,9}\}$
$\mathscr{C}_2 = 1$	$\{s_{4,5}, s_{4,6}, s_{5,6}\}$
$\mathscr{C}_3 = 1$	$\{s_{1,10}\}$
$\mathscr{C}_4 = 1$	$\{s_{2,10}\}$
$\mathscr{C}_5 = 1$	$\{s_{3,10}\}$
$\mathscr{C}_6 = 1$	$\{s_{4,10}\}$
$\mathscr{C}_7 = 1$	$\{s_{5,10}\}$
$\mathscr{C}_8 = 1$	$\{s_{6,10}\}$
$\mathscr{C}_9 = 1$	$\{s_{7,10}\}$
$\mathscr{C}_{10} = 1$	$\{s_{8,10}\}$
$\mathscr{C}_{11} = 1$	$\{s_{9,10}\}$