Rewiring of scale-free networks vs. degree correlation properties

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We present results obtained with new algorithms which use Newman's network rewiring to achieve some target degree correlations, or to obtain assortative or disassortative mixing. In particular, Newman's rewiring has been tested by reconstructing Barabasi-Albert networks, and this procedure gives useful insights about key issues of the theory of scale-free networks, like the degree distribution of the hubs and the role of higher-order correlations. Furthermore, we have proven a new relation between the variation of r (Newman assortativity coefficient) and K (average nearest neighbor degree) valid in any degree-conserving rewiring. – This presentation is based on our works [1] and [2].

The working principle of the Newman rewiring [3] is the following. After choosing at random in the list of the links two links (a,b) and (c,d), between nodes a,b,c,d, one computes the probability E1 of these links according to the "target" correlation matrix. One then computes the probability E2 for the exchanged links, i.e. for the couple (a,c) and (b,d) and applies a sort of Metropolis-Monte Carlo criterion: if E2>E1, then the rewiring is performed with probability 1, otherwise it is performed with a probability proportional to the difference (E1-E2). These steps are repeated for a large number of times, typically at least 103 times the total number N of nodes in the network. Clearly the final product at equilibrium is a statistical ensemble of networks displaying the target correlations in an average sense.

In the reconstruction of Barabasi-Albert networks we have used the general expression for their degree correlations recently computed by Fotohui and Rabbat. For networks "BA-1" (single preferential attachment) the resulting number of isolated pairs is always zero, because the condition P(1|1) = 0 is enforced in an effective way. The size of the giant (connected) component is about 0.69 ± 0.01. Most of the disconnected small components are triples, whose origin is quite interesting. The correlation P(2|1) is non zero for BA-1 networks. In fact, in the growth process with preferential attachment, connected tails of variable length can arise, in which the last node contributes to the correlation P(2|1) and the intermediate nodes contribute to the correlation P(2|2). When the network is re-constructed in the configuration model, isolated triples arise, because the non-vanishing conditional probability P(2|1) allows to attach two nodes of degree 1 to a central node of degree 2 "without knowing" that on the other side of this central node there is no connection to the giant component. This is a simple demonstration of the general fact that the knowledge of the degree distribution and two-point correlations is insufficient to completely characterize a network. For BA degree distributions with multiple attachment the Newman rewiring always generates a fully connected network (giant component equal to 1).

References

[1] M.L. Bertotti, G. Modanese, The configuration model for Barabasi-Albert networks, Appl. Net. Sci. 4, 1 (2019).
[2] M.L. Bertotti, G. Modanese, Network rewiring in the r-K plane, Entropy 22, 653 (2020).
[3] M. Newman, Mixing patterns in networks, Phys. Rev. E 67, 26 (2003).