Invariant and dually flat information geometric structure for deformed exponential families

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Information geometry is a mathematical framework that studies the geometry of the space of probability density functions and the geometry of statistical manifolds. It is an interdisciplinary field that combines differential geometry, statistics, information theory, machine learning theory and statistical physics. Information geometry is a research field that can be called one of the junctions of mathematical sciences.

In information geometry, there are two important concepts for understanding the statistical manifold of probability density functions: invariance and dual flatness. Invariance is the property that the geometric structure of the statistical manifold formed by the family of probability density functions does not change under the transformation of random variables. For example, if we consider the logarithm of a random variable that follows the normal distribution, it follows the lognormal distribution. In this case, the geometric structure determined by the family of normal distributions and by the family of lognormal distributions are exactly the same. Invariance is a fundamental concept in physics and it seems to be related to the symmetry of physical systems. Dual flatness is the property that the Riemannian metric of a statistical manifold is not Euclidean, but the dual affine connections are flat. It is known that an exponential family, including the family of the normal distributions, is a dually flat space. Noteworthy properties include the potential of the Fisher metric being negative entropy and the dual coordinate system obtained from the Legendre transformation.

Information geometry of non-additive statistical physics has focused mainly on dual flatness. Different from the usual exponential family, there are several natural dually flat structures in deformed exponential families. We have been able to clearly discuss the information geometric structures for deformed exponential families: the negative non-additive entropy can be regarded as the potential of the Riemannian metric, and divergence function can be described using escort expectations. In this talk, we consider invariant and dually flat geometric structures in deformed exponential families. Since this structure cannot be calculated directly from the non-additive entropy, it has not been studied. In this talk, we propose a new type of divergence that induces an invariant and a dually flat structure. We expect this result to be a useful development for both information geometry and non-additive statistical physics.