Solvable model for the decline of unconventional oil and gas

Michael Marder
The University of Texas at Austin, Austin, United States

The use of horizontal drilling and hydrofracturing may represent the final stages in the extraction of hydrocarbons to meet exponentially growing world energy demands. Therefore an accurate theory for the decline of oil and gas production from unconventional wells should be of value. We found that the production rate of unconventional wells is described quite accurately as the sum of two universal curves. The first describes the early stage of production, while the second describes a stage of production so late that few wells have yet entered it. In the universal curve for the first phase, production falls first as the square root of time, and then drops exponentially. In the universal curve for the second phase the exponential drop slows, and converts to a much lower rate of decline that goes as one over a log of time. One of the implications of the late-time universal curve is that over times on the order of 30 years existing wells should yield around 30% more gas than would be expected from extrapolating behavior from the first regime. That production data fit the universal curves so well is a puzzle because the fracture networks that collect oil and gas must be highly varied and highly complex. To address this problem, we posed and then solved an analytical model for gas diffusion to complex structures. Some technical problems had to be solved in order to provide the exact solution. The method used was a T matrix formalism on a lattice, and it required computation of lattice Green's functions at arbitrary locations for energy in the complex plane, and for hopping numbers on the order of 100. The recursion relations that provide the fastest and most reliable way to find lattice Green's functions are completely unstable. This difficulty was overcome with use of arbitrary-precision arithmetic. Once exact solutions for the lattice mode of diffusion to fractured networks were in hand, it became evident that most details of the fracture network are irrelevant for the history of gas production. It takes only a few parameters, each with physical meaning, to describe the production history, and other details of the network geometry, in most cases fade away.