Optical thermodynamics of nonlinear systems

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In recent years, considerable effort has been devoted to the study of nonlinear highly multimoded optical systems [1]. The physical motivation behind these theoretical and experimental efforts has been the search for highpower optical sources that has been enabled by a sequence of new developments in multimode technologies pertaining to both guided wave structures and optical cavities. Understanding and predicting the complex nonlinear response of such systems especially when hundreds or thousands of modes are involved, is a challenging task. In the best case, all relevant approaches are mostly based on complicated nonlinear optical simulations, that make the description of realistic multimode fibers a formidable task. Thus a theory that explains and predicts such a complex behavior is still missing. Quite recently however, a self-consistent theoretical framework has emerged, what we call ”Optical Thermodynamics” [2–6]. In particular, optical thermodynamic theory is capable of describing such complex phenomena by means of thermodynamics of the system's supermodes. A complete set of thermodynamical variables was determined and thus was able to describe and accurately predict the equilibrium behavior of the multimoded system. The equation of state, the entropy and the Rayleigh-Jeans modal occupancies distribution was derived axiomatically either on thermodynamical grounds [2–4] or equivalently on statistical mechanical foundations [5, 6]. Such an approach is universal since it can be applied to any weakly nonlinear optical multimode system of finite number of supermodes that involves a finite number of conserved quantities. We can derive the fundamental relations that govern the grand canonical ensemble through maximization of the Gibbs entropy at equilibrium. In this classical picture of statistical photo-mechanics, we obtain analytical expressions for the probability distribution, the grand partition function, and the relevant thermodynamic potentials. The first part of the talk is devoted to the understanding of the role of equilibrium fluctuations and the second part to develop a non-equilibrium description of the system. In order to achieve our first goal we are going to rely on the grand canonical formalism [5] and directly calculate the relevant fluctuations based on the grand partition function expression. For states far from equilibrium, we develop a Langevin type of approach for the projection modal coefficients and derive effective stochastic equations that govern every supermode. Our analytical expressions are compared with direct numerical results of system-bath simulations, in all cases, and the agreement is excellent. In conclusion, by means of statistical mechanics, we have established a solid foundation for the optical thermodynamics for equilibrium and non-equilibrium states. This formulation was carried out in the grand canonical ensemble picture and is applicable to any nonlinear arrangement involving conserved quantities such as the power, Hamiltonian, and a finite set of distinguishable modes. The equilibrium expressions for the fluctuations of power, Hamiltonian and modal occupancy number were found in excellent agreement with direct bath-system simulations. Even more interestingly, we were able to apply a Langevin type of formalism in order to understand the non-equilibrium behavior of our system. Our results universally apply to any other weakly nonlinear highly multimoded bosonic arrangement.

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