A sunburst of solutions in the continuous negative perceptron

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The landscape properties of high-dimensional constraint satisfaction problems (CSPs) can completely determine the type of configurations that can be efficiently sampled from their space of solutions. In recent years, empirical studies on the landscape of neural networks have shown that low-lying configurations are often found in complex connected structures, where zero-energy paths between pairs of distant solutions can be constructed. In the present work, we investigate the connectivity of solutions in the negative perceptron, a linear neural network model and a prototype of a non-convex continuous CSP. We introduce a novel analytical method for characterizing the typical energy barriers between groups of configurations sampled from the zero-temperature measure of the problem. We find that, despite the overall non-convexity of the space of solutions, below a critical density of constraints $\alpha_\star$, the geodesic path between any solution and the robust solutions of the problem, located in the interior of the solution space, remains strictly zero-energy. We study the shape and the anisotropy of the connected space of solutions, and numerically characterize a sharp transition where the simple connectivity property breaks down.