Application of g-subdiffusion equations with the fractional Caputo time derivative with respect to another function in modeling of anomalous diffusion processes.

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We consider a g-subdiffusion equation with a fractional Caputo time derivative with respect to another function g [1-6]. We derive the equation by means of a modified continuous time random walk model (the g-CTRW model) [2]. An interpretation of g-subdiffusion is that it is the ordinary subdiffusion process with a changed time scale controlled by a deterministic function g. The g–subdiffusion equation is shown to be quite general. Changing of a time scale can lead to changes in diffusion parameters and/or in the type of diffusion. Thus, the g-subdiffusion equation offers different possibilities for modeling diffusion, such as a process in which a type of diffusion evolves continuously over time. The g-subdiffusion equation can be solved by means of the g-Laplace transform method; the g-Laplace transform is a generalization of the "ordinary" Laplace transform [1,2]. By defining the function g appropriately, the equation describes a smooth transition from "ordinary" subdiffusion to other processes such as ultraslow diffusion [1], "ordinary" subdiffusion with changed parameters [3], and superdiffusion [6]. In the transition from subdiffusion to superdiffusion, the fundamental solution (the Green's function) for the g-subdiffusion equation takes the form of Green's function for superdiffusion described by the equation with the fractional Riesz derivative with respect to a space variable. We conclude that for a sufficiently long time the g–subdiffusion equation describes superdiffusion well, despite a different stochastic interpretation of the processes. Then, paradoxically, a subdiffusion equation with a fractional time derivative describes superdiffusion. The superdiffusive effect is achieved here not by making anomalously long jumps by a diffusing particle, but by greatly increasing the particle jump frequency. Some methods used in modeling of "ordinary" subdiffusion processes, such as the derivation of local boundary conditions at a thin partially permeable membrane, can be used to model g–subdiffusion processes, even if this process is interpreted as superdiffusion. We also use the g-subdiffusion equation to describe anomalous diffusion of antibiotic (colistin) in a system consisting of packed gel (alginate) beads immersed in water. Experimental results show that this process cannot be described by the "ordinary" subdiffusion equation with constant parameters. However, the g-subdiffusion equation with the function g derived from empirical data can be used to describe this process which is interpreted as subdiffusion with changing subdiffusion parameter (exponent) [4]. In addition, there is considered g-subdiffusion process of molecules that can be annihilated with a constant probability independent of time; molecule anihilation equation also includes the g-Caputo fractional derivative [5]. Such g-subdiffusion-anihilation model can be used to describe antibiotic diffusion in a bacterial biofilm.

References

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