A game-theory-inspired reinvestigation of the Blume–Capel model

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Two-player interactions in evolutionary game theory are typically represented by matrix games. Recent research into the linear decomposition of square matrices has revealed a complete set of payoff matrices that define just four fundamentally different, archetypal interaction situations: elementary self- and cross-dependent, coordination, and cyclic-dominance games [1]. The general, n-strategy elementary coordination game and its combinations with a self-dependent game component that retains its symmetry [2] – when played according to the logit strategy update rule by numerous players located at the sites of a square lattice against their nearest neighbours – can be shown to be equivalent to a Blume–Capel model with a temperature-dependent crystal-field coupling through a consistent bunching of its n-2 neutral strategies into a single strategy affected by an additional noise-level-dependent self-dependent game component, making the Blume–Capel model an ideal starting point for the systematic study of the interplay of elementary games. Here, I corroborate these analytic findings with numerical results obtained using the tensor renormalization group method introduced by Michael Levin and Cody P. Nave [3] by reinvestigating the phase transitions of the Blume–Capel model along the directions that correspond to elementary coordination games. The results are in good agreement with expectations based on the literature of the Blume-Capel model [4]: They indicate that the phase transitions do indeed occur at the expected locations and have critical exponents that are consistent with those of the two-dimensional Ising model along the line of continuous phase transitions and vanish for jump-like changes where first-order transitions are expected. I also extend my calculations to the less frequently studied case of when the Blume–Capel model is subjected to a homogeneous external magnetic field. In keeping with earlier mean-field approximation results, first-order transitions can seemingly remain of the first order for small enough magnetic fields and only become smoothed out, unlike the continuous transitions, for stronger magnetic fields.

References

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