Nonlinear and non-local FPK equation for probabilistic response of nonlinear systems under Gaussian colored noise excitation

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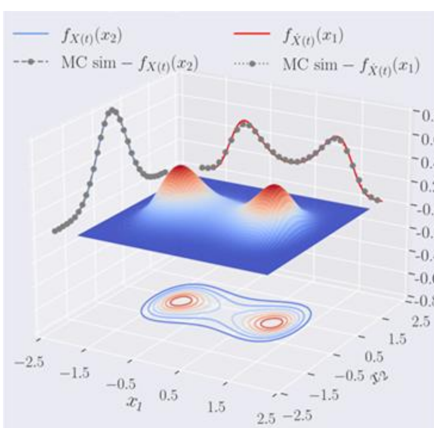
The statistical (probabilistic) response of nonlinear Random Dynamical Systems (RDS) excited by Gaussian colored noise is an important problem, arising in many applications in Physics and Technology. The loss of the Markovian character of the response, complicates the solution procedure and calls for special solution techniques. The main methods in use for treating such problems are: i) embedding the problem in a higher-dimensional Markovian one, ii) asymptotic techniques, as e.g. the unified colored noise approximation [1], [2], which works well in a restricted domain of the parameter space (correlation time and noise intensity). Obvious drawbacks of these methods are, the increase of degrees of freedom, in the first approach, and the restricted applicability, in the second. In the present paper we present a different approach, deriving a novel efficient FPK-like equation, for the first-order probability density function (pdf), which embodies non-Markovian effects through nonlinear and history-dependent terms. A first announcement of the theory, without numerical results, has been given in [3], [4]. This equation generalizes a similar one developed for the case of scalar Random Differential Equations [4], [5] The derivation is based on the Stochastic-Liouville equation (SLE) of the RDS, which is a non-local equation containing averages over nonlinear expressions of the time history of the response. This is further reformulated by applying the Extended Novikov-Furutsu Theorem [6], which leads to another form of SLE containing averages over the transition matrices of the variational problems associated with the given RDS (time-varying linear IVPs). This equation is exact, yet non-closed. The novel closure applied to this equation is based on a decomposition of response in its local (in time) mean value and its fluctuation around it. Then, we apply a current time approximation to the fluctuating part, keeping in full the effect of the local mean value, which is expressed as a time-history term of some moments of the response. In that way we obtain our final, efficient FPK-like equation, which is closed, yet nonlinear and nonlocal. This equation reduces to the usual multidimensional FPK equation in the case of delta-correlated excitation. Further, it reduces to a linear PDE in the case of a linear RDS, providing the well-known Gaussian solution. Numerical results for the new nonlinear, nonlocal equation are provided for a bistable Duffing oscillator under Gaussian excitation with correlation time 0.92. The obtained pdf compares well with Monde Carlo simulations.



Steady state pdf - new FPK solution

Duffing oscillator, $\ddot{X}(t) + \zeta\,\dot{X}(t) - X(t) + X^3(t) = \xi\,\Xi(t)$ (non-dim. form), where $\zeta = 0.25$ and $\xi = 0.6$. The correlation time of the Gaussian excitation $\Xi(t)$ is $\tau_{cor}^{\Xi} = 0.92$.

References

[1] P. Hanggi, P. Jung, Adv. Chem. Phys., 89 (1995).
[2] W.-L. Duan, H. Fang, Physica A, 555, 124624 (2020).
[3] K. Mamis, G. Athanassoulis, K. E. Papadopoulos, Procedia Comput Sci, 136, 249 (2018).
[4] K. I. Mamis, PhD Thesis, National Technical University of Athens, 2020.
[5] K. I. Mamis, G. A. Athanassoulis, Z. G. Kapelonis, Proceedings of the Royal Society A (2019).
[6] G. A. Athanassoulis, K. I. Mamis, Phys Scr, 94, 11 (2019).