Correlated percolation of sites not removed by a random walker in 2≤d≤6 dimensions

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Mechanical properties of a gel which is degraded by few enzymes that travel through it and brake crosslinks can be modeled by the removal of sites on a lattice by a meandering random walk (RW) [1]. We studied [2] a percolation problem on a d-dimensional hypercubic lattice of size L^d with periodic boundary conditions, where a RW of $N=uL^d$ steps removed sites. Sites not visited by the RW have correlation that decays with the distance r as $1/r^{d-2}$. [However, crossing the (periodic) boundaries breaks the RW into almost uncorrelated segments.] In d≥3 for large L the RW length parameter u is simply related to the fraction of unvisited sites $p=\exp(-A_d\ u)$, where A_d are known constants. As u increases (p decreases) the critical value u_c is reached, beyond which the unvisited sites no longer percolate. For 3≤d≤6 we found that $u_c \simeq 3$. Close to the percolation threshold the correlation length (mean linear size of a finite cluster) diverges as $|u-u\_c|^{-\nu}$, with ν=1/(d-2), as predicted by the theory of correlated percolation [3] This and other critical exponents (including the exponent of conductivity) differ significantly from the case of the uncorrelated (Bernoulli) percolation, but, with increasing d, their values approach the same mean field values at d=6.

In d=2 the percolation probability is a smooth function of u and the problem has no percolation threshold, because the fractal dimension of the RW coincides with d. Nevertheless, the system has scaling properties [4], where the system size L plays a role resembling correlation length in higher-dimensional cases: Boundaries of the clusters of unvisited sites have fractal dimension 4/3, while the mean cluster size (mass) is proportional to L^2. Density of clusters of size s scales as s^{-t}, while the volume fraction occupied by k-th largest cluster scales as k^{-q}. We numerically measured the $t \simeq 1.8$ and $q \simeq 1.2$, which are outside the range of allowed values for such exponents, and we suggest that these are effective numbers that converge to their ultimate values (2 and 1, respectively) as 1/ln L. We provide a heuristic argument for their $L=\infty$ values.

References

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