Super-additivity, generalized concavity and quasi-homogeneity in non-additive systems

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The properties of an entropy function, such as super-additivity, concavity, and homogeneity, set up the basis for the maximum entropy principle, which arises from the second law of thermodynamics [1]. According to the maximum entropy principle, the equilibrium state of a composed system is found to be the one that maximizes the sum of entropies of its subsystems, and the equilibrium conditions are found by the zeroth law of thermodynamics [2]. The standard results show that the satisfaction of two of the properties implies the third one (see [3] and references therein), which sets up the criterion for the thermodynamic consistency of the chosen entropy form. However, these results are applicable only to additive systems.

On the other hand, in the case of non-additive systems, which are common in the presence of long-range interactions [4] and in black hole thermodynamics [5], the entropy is not a homogenous, but a quasi-homogenous function. Although some fundamental results, such as the zeroth law of thermodynamics and the Gibbs-Duhamel relationship, have already been established for quasi-homogenous systems [5], the relationships between quasi-homogeneity, concavity, and super-additivity of non-additive systems seem to still be unknown.

In this talk, we will show that quasi-homogeneity and generalized concavity imply the super-additivity of entropy in a non-additive system. The results will be applied to the characterization of the equilibrium of the generalized Landsberg ideal gas, which represents a model of a simple system that violates homogeneity (and consequently super-additivity), while being consistent with ideal gas state equations. Applications of the results to systems with non-additive energy and non-additive volume will also be discussed. Finally, we will discuss the role of non-additive entropies [6] in the thermodynamics of quasi-homogenous systems.

References

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