Hidden geometry of brain dynamics revealed by persistent homology

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Characterizing the pattern of dependences between the dynamics of nodes of complex systems (such as interacting brain regions) by the tools of topological data analysis is an area of rapidly growing interest. It has been recently shown, that topological data analysis is sensitive to brain-disease-related alterations in both the structure of the functional connectivity (instantaneous statistical dependences) and effective connectivity (directed causal interactions), albeit the detectability of these alterations depends of their topological/topographical specificity [1].

In this contribution we shall discuss the practical aspects of utilization of topological data analysis to characterize the hidden geometry of brain dynamics, as well as provide methodological approaches to gain qualitative insights concerning this hidden geometry. In particular, we present an investigation of the underlying curvature of data through the lens of topology [2].

Building upon previous work [Giusti et al., 2015], we employ the tools of Persistent Homology, namely topological features derived from Betti curves. We first investigate the case of random and geometric matrices (distance matrices of points randomly uniformly distributed on manifolds of constant sectional curvature). We consider the three classical models given by the Euclidean space, the sphere, and the hyperbolic space. We show that Betti curves effectively distinguish these spaces. Thus we can use manifolds of constant curvature as comparison models to infer properties of the underlying curvature of manifold underlying real data.

We analyse brain dynamics data (while comparing with financial and climate data examples) and observe that their associated topological features appear to emerge from hyperbolic underlying geometry. This result is consistent with the general belief that their underlying data manifold is of non-positive curvature, however we also discuss alternative explanations related to the data sampling and processing steps, as well as more complex possible hidden geometries.

References

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