Strong, weak or no balance? Testing structural hypotheses against real networks

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The abundance of data about social, economic and political relationships has opened an era in which social theories can be tested against empirical evidence, allowing human behaviour to be analyzed just as any other natural phenomenon. The present contribution focuses on balance theory, stating that social actors tend to avoid establishing `frustrated' network configurations where positive (friendly) and negative (hostile) interactions conflict with each other [1,2]. To make this theory testable, one needs 1) a proper representation of social networks, 2) a definition of frustrated configurations and 3) a set of null models with respect to which the amount of frustration in a given, real network can be checked for statistical significance. The first two ingredients have been already explored comprehensively: social interactions can be represented via signed graphs (where edges can be positive, negative or absent) and frustrated configurations are defined (in strong form) as having cycles with an odd number of negative links (although a weaker definition exists as well). The third ingredient, however, is way less developed in current research, since the existing null models cannot take into account the heterogeneity of individual actors, i.e. their different tendencies to establish positive and negative interactions. To reduce this gap, we extend the Exponential Random Graphs framework to binary, undirected, signed networks with both global constraints (overall numbers of positive, negative and missing links) and local constraints (node-specific numbers of positive and negative links). Moreover, we define two variants for each benchmark: one where the topology is kept fixed and one where it is left to vary along with the edge signs. When applied to real networks, the new null models show that the level of frustration crucially depends on (at least) three factors: 1) the nature of the data (e.g. biological VS socio-political networks), 2) the measure adopted to quantify balance (e.g. weak VS strong form), 3) the null model employed for the analysis (e.g. homogeneous VS heterogeneous, fixed VS varying topology). As the attached figure shows, the analysis of triangles reveals that homogeneous null models with global constraints (e.g. the Signed Random Graph Model, SRGM) tend to favour the weak version of balance theory, according to which only the triangle with one negative link (depicted in green) should be under-represented in real, social and political networks (as a comparison, biological networks are instead found to be significantly frustrated). On the other hand, heterogeneous null models with local constraints (e.g. the Signed Configuration Model, SCM) tend to favour the strong version of balance theory, according to which also the triangle with all negative links (in purple) should be under-represented in real, social networks (again, biological networks behave differently).

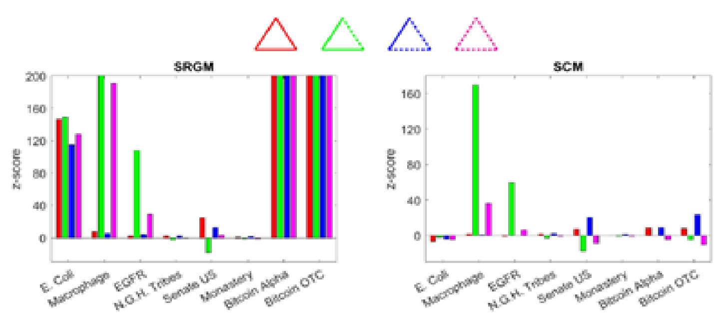

FIG. 1: z-scores of triadic motifs for three biological networks (*E. Coli*, *Macrophage*, *EGFR*) and three social and political networks (*N.G.H. Tribes*, *Senate US*, *Monastery*). While homogeneous null models (SRGM, left panel) support weak balance theory on social and political networks, heterogeneous ones (SCM, right panel) tend to favour strong balance theory. As a comparison, biological networks show a different behaviour.

References

[1] Heider, J. Psychol., 21, 107 (1946).
[2] Cartwright et al., Psychol.Rev., 63, 277 (1956).