Can memory hysteresis in a neural network judge the continuity/discontinuity of a phase transition?

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The application of machine learning (ML) has been extended quite rapidly over many disciplines [1] and is also pervasive for solving problems in a wide area of physics. Detecting a phase transition using ML is an intriguing trial [2]. The success prompted us to apply the nonequilibrium relaxation idea to the problem to show that phase-transition detection could be finished early in the learning processes [3]. These works utilized that the sum of the weights between neurons in a convolutional neural network (CNN) behaves like an order parameter.

Here, we focus on not detection but judging the (dis)continuity of a phase transition as another application of ML. The following memory hysteresis experienced in everyday life inspired us to undertake the present investigation. When we watch a series of continuously varying pictures, we recognize that the point at which the picture undoubtedly changes differs if we see the sequence in reverse order. The degree of this hysteresis depends on the degree of continuity; the better the continuity, the more significant the hysteresis. If there is a sudden change in the series, we may recognize the point uniquely regardless of the direction of variation. Because a CNN is implemented to mimic the functionality of a biological neural network, the degree of (dis)continuity would be reflected in the hysteretic behavior of the weights in a CNN in back-and-forth learning processes, i.e., when a CNN learns the phase-transition patterns in increasing (decreasing) order of the labels.

We employed the two-dimensional Ising and q-state Potts (q=3 through 6) models. Equilibrium spin configurations above and below the critical temperature were generated as a function of the external field for the Ising model; those at various temperatures around the critical point were prepared for the Potts model. The external field and temperature constitute the labels. We let our CNN [4] learn the difference between the configurations with adjacent labels sequentially in increasing order (forward learning) and then decreasing order (backward learning) of the labels. The Potts' order was simplified in some way for CNN to capture the difference quickly. We monitored the weights connected to the output layer in forward and backward learning.

The memory hysteresis, i.e., hysteresis in the CNN's weights, in back-and-forth learning appeared for the Ising crossover transition and the transitions of the Potts model with q=3, 4. No hysteresis was recognized for the Ising transition below the critical point and the transition of the q=6 Potts model. Interestingly, the weights behaved marginally for a weak first-order transition in the q=5 Potts model. The (dis)continuity of a phase transition could thus be adequately judged by the aspect of memory hysteresis of a CNN through sequential learning of the differences in the patterns around the transition point.

References

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