Nonlinear response in dilute colloidal suspensions beyond the fluctuation-dissipation theorem

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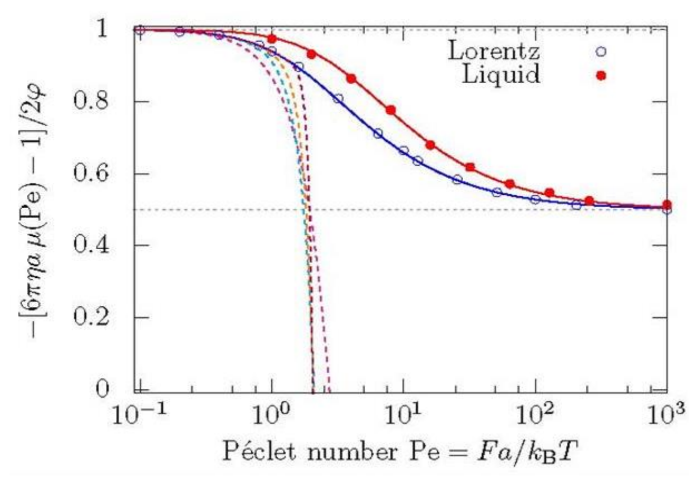
In a microrheological experiment the thermal or forced motion of a colloidal particle is monitored to obtain information on mechanical properties of the surroundings [1]. While the linear response is well-characterized in terms of the fluctuation-dissipation theorem, few exact results are available for strong driving. Here, we consider the time-dependent velocity v(t, F) of a colloidal particle immersed in a dilute suspension of hard spheres in response to switching on a finite constant force F at time zero. The dimensionless number quantifying the strength of the driving is the Peclet number Pe = F a/kB T, where a denotes the radius of the hard spheres. Our main quantities of interest are the time-dependent mobility μ(t,Pe) = v(t,Pe)/F and its approach to the stationary state mobility μ(Pe)=μ(t -> ∞, Pe). A stationary state solution for the mobility exact in first order of the packing fraction ϕ has been established earlier in terms of a power-series expansion [2]:

$$6 \pi \eta \; a \mu(Pe) = 1 - 2 \phi [1 - Pe^{2/30} + |Pe|^{3/64} + O(Pe^4)],$$

where η denotes the shear viscosity. Here, we extend this results to the case of arbitrarily strong driving including the complete time-dependence of the response upon switching on the force [3]. In the stationary state, our analytic solution recovers the anticipated limit for strong driving (see figure) 6πηa μ(Pe) = 1-ϕ and captures the nonlinear response in first order of the packing fraction for any strength of the force. The time-dependent drift velocity approaches its stationary-state value exponentially fast for arbitrarily small driving in striking contrast to the power-law prediction of linear response encoded in the long-time tails of the velocity autocorrelation function. We show that the stationary-state behavior depends nonanalytically on the driving force and connect this behavior to the persistent correlations in the equilibrium state. We argue that this relation holds generically. Furthermore, we elaborate that the fluctuations in the direction of the force display transient superdiffusive behavior.



References

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[3] S. Leitmann, S. Mandal, M. Fuchs, A.M. Puertas, T. Franosch, Phys. Rev. Fluids 3 (10), 103301 (2018).