Temperature distribution in finite systems: Application to the one-dimensional Ising chain

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The interactions of small systems with a finite environment in thermodynamics, display an interesting statistical behavior, similar to complex non-equilibrium systems and recently gained relevance for technological applications. In the case of a small system with a finite environment, unlike standard thermodynamic treatment of systems, the subsystem presents energy fluctuations and temperature uncertainty. Conceptually, the idea of uncertainty is often confused with that of fluctuations, the latter being usually related to systems that present a particular dynamics, on the other hand, uncertainty appears in both static and dynamical models of systems. Moreover, in theories such as superstatistics, there are cases in which the uncertainty in the inverse temperature β cannot be understood as fluctuations of an observable quantity, but instead can only be treated as lack of information about an unknown, constant parameter to be inferred. In the literature, the study of small systems with finite environment has been pursued by several authors either using the traditional techniques from statistical mechanics, or proposing suitable generalizations of Boltzmann-Gibbs statistical mechanics. A particularly interesting way to treat these types of systems is by means of the theory of superstatistics, that generalizes Boltzmann-Gibbs statistical mechanics and aims to explain the statistics of different types of complex systems, such as plasmas and self-gravitating systems. Only lately, some applications to the thermodynamics of small systems characterized by short-range interactions have been presented.

Accordingly, in this work we propose to use the properties of the ensemble and the microcanonical inverse temperatures to explore and study the necessary conditions for superstatistics in the context of small systems. As a particular case, we describe the behavior of a one-dimensional Ising subsystem as a part of an isolated Ising chain, as recently reported. Under the constraints that the superstatistical theory imposes on the two temperature functions, ensemble and microcanonical, we show that the non-canonical distribution that describes the Ising subsystem is not consistent with this theory, despite having temperature uncertainty. Our results hint at a new framework for dealing with regions of microcanonical systems with positive heat capacity, which should be described by some new class of statistical ensembles outside superstatistics but still preserving the notion of temperature uncertainty.