Different kinds of localization in complex networks

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We discuss localization effects observed in various matrix representations of complex networks and in processes taking place on networks [1]. The localization is indicated by a non-vanishing inverse participation ratio in large networks. The first basic example of localization is demonstrated by the quenched mean-field theory for the epidemic SIS model on a complex network with a hub or a cluster with high connectivity. This problem is reduced to the localization of the principal eigenvalue of the adjacency matrix of a network [2]. The explicit solution was found for a sparse random regular graphs and for Erdős-Rényi random graph with a hub. For a disease spreading, localization results in an island of infected vertices in a range of transmission rates below the endemic epidemic threshold.

Localization of the principal eigenvalues of the adjacency and Laplacian matrices of complex networks with a hub significantly hinders community detection. A non-backtracking matrix is widely used as a remedy against localization on isolated hubs. Nonetheless, the principal eigenvalues of this matrices still can be localized on clusters. We describe the explicit solution for localization of the principal eigenvector of the non-backtracking matrix on an arbitrary finite graph inserted into an arbitrary infinite tree-like network [3] derived in the framework of the non-backtracking expansion approach [4].

Importantly, the quenched mean-field approximation neglects the absorbing state in the SIS model and fluctuations, due to which a finite number of infected vertices all will finally become susceptible due to fluctuations. Hence this solution with a finite number of infective vertices is meaningful only in the quasi-stationary, metastable state, and so localization on a hub in the SIS model is actually metastable [5]. This trouble disappears for localization on a large cluster still containing a vanishingly small fraction of an in infinite network. We outline the features and applications of the metastable localization.

References

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