Scale-free network architectures generated by nonlinear preferential attachment

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The preferential attachment mechanism is one of possible origins of power-law degree distributions $P(q) \propto q^{-\gamma}$ in complex networks. New vertices attach to existing ones with a probability proportional to a preference function of their degrees, $f(q)$. It is widely known that linear preference functions provide scale-free networks [1], while power-law preference functions with exponents distinct from 1 result in non-scale-free architectures [2]. It was noticed however that a linear asymptotics of the preference function turns out to be sufficient to get a power-law degree distribution, and that, for a recursive tree, even distorting a linear preference function at the smallest degree $q^* = 1$, one can vary the exponent γ of the degree distribution within the range $(2, \infty)$ [2]. Remarkably, a small distortion of this kind produces a strong effect on γ.

We study the effect of the distortion of a linear preference function $f(q) = q + a$ at single degrees q^* on a shape of the degree distributions of recursive trees. Without this distortion, exponent $\gamma = 3 + a$, where $-1 < a < \infty$. We find the explicit solution of the stationary limit of the rate equations for degree distributions and show that the effect of this distortion decreases with q^*.

For $q^* = 2$, varying $f(2)$ in the range $(0, \infty)$, we obtain exponent γ in the range $(2, \gamma_{max}(a))$, where $\gamma_{max}(a) \cong [(1 + \sqrt{5})/2]a$ for large a. The factor $(1 + \sqrt{5})/2 = 1.618..$. is the golden ratio. Further, for $q^* =$, varying $f(3)$ in the range $(0, \infty)$, we obtain exponent γ in the range $(\gamma_{min}(a), \gamma_{max}(a))$, where $\gamma_{min}(a) \cong 0.618 \ldots a$ and $\gamma_{max}(a) \cong 1.247 \ldots a$ for large a.

For large q^ $*$, the range of possible values of exponent γ becomes quite narrow, converging to $\gamma = 3 + a$ as $q^* \to \infty$. The limiting minimum value $\gamma_{min} = 2$ is approachable only in the narrow range of a close to -1, namely for $-1 < a < -1 + 1/(\ln q^* + \gamma_e)$, where $\gamma_e = 0.577..$. is Euler's constant.

These results indicate the range of scale-free architectures produced by such manipulations of a preference function.

References

[1] S. N. Dorogovtsev, J. F. F. Mendes, The Nature of Complex Networks (Oxford University Press, Oxford, 2022).
[2] P. L. Krapivsky, S. Redner, Organization of growing random networks, Phys. Rev. E 63, 066123 (2001).