Non-Gaussian models of financial returns: comparisons and applications

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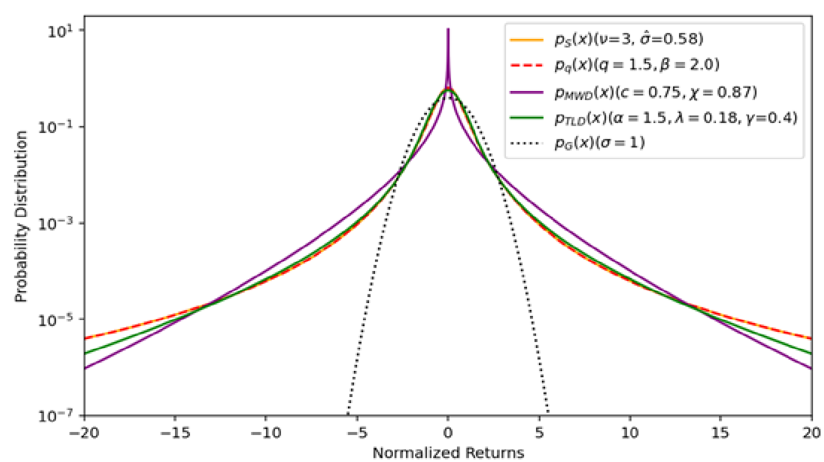
To describe the probability distribution function of returns, the normal distribution was the first suggestion. It is still a landmark and is widely used in finance, yet it is based on some hypotheses which are not empirically verified. In fact, the analysis of empirical data clearly shows that high-frequency returns follows a leptokurtic, heavy-tailed shape. Different non-Gaussian models have then been proposed to account for these features, such as the Truncated Lévy, the Student's t, the q-Gaussian and the Modified Weibull distributions. There is no consensus on what class of probability distributions should be adopted to describe financial returns, since the different models have demonstrated, to varying extent, an ability to reproduce empirically observed stylized facts. In order to provide some clarity, we perform a comparative study of the aforementioned non-Gaussian models. To introduce a coherent framework, each distribution is reported to zero mean (to model price changes with subtracted average returns), unit variance and evaluated over a bounded range (to mimic realistic outliers). The shape parameter of each model is chosen according to values of the literature which provide a good fit to empirical data, whereas the scale parameter is obtained with the constraint of unit variance. Given the above setting, we are able to implement a consistent comparison of the four models, which show a similar leptokurtic shape and a quite good agreement along the tails. To further investigate the outliers behavior, we focus on the complementary distribution functions and on the kurtosis; similarities emerge in these cases as well. By using different Monte Carlo algorithms, we generate large samples of non-Gaussian random numbers as synthetic data representations of financial fluctuations, in order to compare their statistical properties and to simulate their dynamical evolution, highlighting the differences with the normal scenario. We also present a first application to option pricing, by considering both plain vanilla and path-dependent options for a comparison of Gaussian and non-Gaussian fluctuations. To this aim, we adopt the standard risk-neutral approach to derivative pricing. We observe differences between the two scenarios in the limit of short maturities, while, as the maturity increases, our results gradually converge to the Gaussian predictions as a consequence of the limit behavior of the fat-tailed distributions.



References

[1] F. De Domenico, G. Livan, G. Montagna, O. Nicrosini, Modeling and Simulation of Financial Returns under Non-Gaussian Distributions, arXiv:2302.02769 [q-fin.ST]