Equivalence of solitonic solutions in a neuron chain and single neuron delay differential equations

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In recent years, the refinement of the technologies applied to neural imaging is making data available at finer and finer resolutions, while the amount and variety of such data has been continuously increasing [1]. These two aspects create a need for interpretable models that connect phenomena across several scales, and thus an opportunity to tackle these problems with tools from Statistical Physics. Mesoscopic models can be formulated in the framework of Dynamical Systems on Graphs, where a set of nodes, each endowed with an internal dynamics reproducing the spiking properties of neurons, is coupled according to a directed network structure. The dynamical properties of such systems are determined by the interplay of structure and dynamics, so that nonstationary stable states and emergent properties exist. For these models cycles play an important role, both as characterising structural features and as the possible origin of feedback mechanisms and self-organisation. Considering, in particular, applications to the modelling of neural phenomena, we study a periodic chain of FitzHugh-Nagumo neurons with a unidirectional gap-junction coupling. We show the existence of a regime of solitonic travelling waves, emerging through a transition that does not destabilise the fixed point of the system (corresponding to a state where all neurons are quiescent, with no activity on the network). We perform extensive numerical simulations to characterise the dependence of the solutions on the model parameters. In this regime we show that the solution of the chain is equal to the periodic solution of a single FitzHugh-Nagumo equation with an explicit time delayed feedback term of the same form of the chain coupling, once a suitable delay is set depending on the solitonic wave speed and chain size. The connection between Delay Differential Equations and spatially extended dynamical systems has already been proposed, although in a different context [2]. In the Delay Differential Equation as well as in the chain, the fixed point stability remains unaltered through the transition. This strong common feature points out to the fact that in both systems the change in the dynamics cannot be caused by a Hopf bifurcation of the stable fixed point, even though some global properties of the two systems are different. Further development of this line of work can consist in the study of two or more interacting cycles, building up to the study of network architectures. This approach can hopefully shed light on the role of cyclic structures in neural phenomena such as the emergence of memory effects, synchronisation, or even, if plasticity is implemented, learning of specific tasks.

References

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