The Fermi Pasta Ulam Tsingou (FPUT) Paradox: The Birth of Nonlinear Science

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In 1953, Enrico Fermi, John Pasta, Stan Ulam, and Mary Tsingou initiated a series of computer studies aimed at exploring how simple, multi-degree of freedom nonlinear mechanical systems obeying reversible deterministic dynamics evolve in time to an equilibrium state describable by statistical mechanics. Their expectation was that this would occur by mixing behavior among the many linear modes. Their intention was then to study more complex nonlinear systems, with the hope of modeling turbulence computationally.

The results of this first study of the so-called Fermi-Pasta-Ulam-Tsingou (FPUT) paradox, which were published in 1955 and characterized by Fermi as a "little discovery," showed instead of the expected mixing of linear modes a striking series of (near) recurrences of the initial state and no evidence of equipartition. This work heralded the beginning of both computational physics and (modern) nonlinear science. In particular, the work marked the first systematic study of a nonlinear system by digital computers ("experimental mathematics") and led directly to the discovery of "solitons," as well as to deep insights into deterministic chaos and statistical mechanics.

In this talk, I will review the original FPUT studies and show how they led to the understanding of two key paradigms of nonlinear science. Specifically, I will show how a continuum approximation to the original discrete system led to the discovery of "solitions" whereas a low-mode approximation led to an early example of "deterministic chaos."

I will close with a brief indication of how the recurrence phenomenon observed by FPUT can be reconciled with mixing, equipartition, and statistical mechanics.