The distribution of cover times of random walks on random regular graphs

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We present analytical results for the distribution of cover (C) times of random walks (RWs) on random regular graphs (RRGs) consisting of N nodes of degree c ≥3 [1]. Starting from a random initial node, at each time step an RW hops into a random neighbor of its previous node. In some of the time steps the RW visits new nodes that have not been visited before, while in other time steps it revisits nodes that have already been visited. As a result, the number of distinct nodes s visited up to time t is typically smaller than t. The cover time T_C is the number of time steps required for the RW to visit every single node in the network at least once.

In order to obtain the distribution of cover times, we first calculate the distribution $P_t(S = s)$ of the number of distinct nodes s visited by an RW up to time t. To this end we derive a master equation for the distribution $P_t(S = s)$. Using the generating function approach we obtain an analytical solution of the master equation. Inserting $s = N$ in the distribution $P_t(S = s)$ one obtains $P_t(S = N)$, which is the probability that the RW has visited all the nodes in the network up to time t. In fact, this coincides with the cumulative probability of the cover times, namely $P(T_C \leq t) = P_t(S = N)$. The tail distribution of cover times is given by $P(T_C > t) = 1 - P(T_C \leq t)$. Therefore, $P(T_C > t) = 1 - P_t(S = N)$. Using the relations above, we obtain the tail distribution of cover times.

In the long time limit the distribution of cover times converges towards a discrete Gumbel distribution, known from extreme value theory. The Gumbel distribution often emerges as the distribution of the maxima among sets of n independent random variables drawn from the same distribution. It is one of the three possible families of extreme value distributions specified by the extreme value theory.

References

[1] I. Tishby, O. Biham, E. Katzav, Analytical results for the distribution of cover times of random walks on random regular graphs, J. Phys. A55, 015003 (2021).